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Construction of an integrable field close to any non-integrable toroidal magnetic field

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Abstract

Using a toroidal coordinate system constructed from a framework of rotational quadratic-flux-minimizing surfaces with rational rotation number, an integrable magnetic field is constructed that is nearby a given non-integrable field. Straight-field-line coordinates for the constructed integrable magnetic field are computed. By considering the original magnetic field to be obtainable by a small perturbation to the integrable magnetic field, island widths can be estimated analytically. Since quadratic-flux-minimizing surfaces coincide with flux surfaces when they exist, the constructed coordinates may be designed to reduce to straight-field-line coordinates for the given magnetic field in regions of good magnetic surfaces. The coordinates are illustrated using a magnetic field from the PIES code. © 1998 Elsevier Science B.V.

1. Introduction

For the containment of plasma by toroidal magnetic fields, it is essential that the magnetic field be as close as possible to integrable. That is, that the magnetic field lines lie approximately on nested toroidal *flux surfaces* [1]. This is because, to lowest order in Larmor radius, particle orbits are tied to magnetic field lines. In such a case, one may conveniently describe particle trajectories using straight-field-line flux coordinates [2,3].

A completely integrable magnetic field is in principle only possible for systems with a continuous symmetry, and in practice errors and three-dimensional effects will result in islands and chaotic regions being formed. Nevertheless, even for such devices as stellarators, the concept of flux surfaces and straight-field-line magnetic coordinates is useful as an approximation, because these fully three-dimensional devices are carefully designed so as to minimize islands and chaotic regions in the vacuum field, either by visually examining Poincaré surfaces of section, or by estimating island strengths quantitatively (e.g., by using the residue method [4]). Alternatively, islands of a given periodicity and phase may be introduced in the vacuum field [5], so as to produce smaller islands at finite pressure due to self-healing.

Thus, we need to consider magnetic fields that are

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non-integrable, but *close* to integrable. For practical purposes it is often sufficient to proceed as if the field were exactly integrable, and to attempt to construct straight-field-line coordinates for the non-integrable field on the basis of this assumption. Although islands and straight-field-line coordinates are in principle incompatible, in practice islands are very small except close to the larger island chains and the assumption of nested flux surfaces and the existence of straight-field-line coordinates is an adequate approximation [6].

However, this approach must break down near low order island chains, and is also unsatisfactory from a numerical analysis point of view because it does not provide a precise criterion for its applicability. Since field-line flow is a Hamiltonian system, one natural approach to developing a more systematic algorithm is to approach the problem from the point of view of Hamiltonian perturbation theory [7], in which the Hamiltonian is assumed to be decomposed into an integrable part plus a small perturbation. Then action-angle coordinates are constructed for the underlying integrable Hamiltonian, and, e.g., the widths of islands in the total system are calculated perturbatively. The concept of flux surface in magnetic field line flow corresponds precisely with that of invariant torus in Hamiltonian dynamical systems theory, so that another benefit of the "integrable plus perturbation" decomposition is that Kolmogorov–Arnol'd–Moser (KAM) theory can be brought to bear on the question of existence of flux surfaces.

However, a major problem with implementing this program in the case of stellarator magnetic fields is that the underlying integrable field is *not* given beforehand. Thus, the first aim of the present paper is to present an algorithm for finding a suitable underlying integrable field close to any given non-integrable field. The second aim is to construct straight-field-line coordinates for the neighbouring integrable field in such a way that they reduce to those constructed by the methods used presently in region of good flux surfaces, for example that used in PIES [6] developed in Ref. [8].

The construction is based upon the concept of quadratic-flux-minimizing surfaces [9,10]. These are surfaces that minimize a suitable functional involving the normal component of the magnetic field with respect to a trial surface. In particular, if a flux surface exists, then it automatically minimizes the quadratic flux functional because the normal component everywhere vanishes on the surface. These surfaces can also be shown to minimize the square of the functional derivative of the *field-line action*, which vanishes on dynamically allowed paths.

The algorithm may be summarized as follows: (1) Construct a curvilinear toroidal coordinate system ρ, θ, ϕ, where ρ is a generalized radial coordinate (zero on the magnetic axis, but otherwise arbitrary), θ is a general poloidal angle and ϕ is a toroidal angle (e.g., the normal geometric one used in cylindrical or spherical coordinates); (2) Choose a set of K rational numbers $\{\iota\!\!\!-_k | k = 1, 2, \ldots, K\}$ within the range of rotational transforms present in the plasma; (3) Construct a set of quadratic-flux-minimizing surfaces $\{\rho = \rho_k(s_k, \theta, \phi)\}$ through the islands corresponding to the rotational transforms $\{\iota\!\!\!-_k\}$; (4) Interpolate to provide a continuous transformation $\rho = \rho(s, \theta, \phi)$ such that each case $s = s_k =$ const defines one of the quadratic-flux-minimizing surfaces constructed in the previous step; (5) Find the contravariant representation of $\boldsymbol{B}$ in the s, θ, ϕ coordinate system; (6) Find the neighbouring integrable field $\bar{\boldsymbol{B}}(s, \theta, \phi)$; (7) Construct straight-field-line coordinates (s, θ_0, ϕ) for $\bar{\boldsymbol{B}}$; (8) Construct the field-line Hamiltonian $\chi(s, \theta_0, \phi)$ for $\boldsymbol{B}$.

Section 2 describes the construction of quadratic-flux-minimizing surfaces and the intermediate coordinate system s, θ, ϕ (Steps (1)–(4) above). Section 3 describes the alterations to the magnetic field required to construct an integrable magnetic field with flux surfaces corresponding to the quadratic-flux-minimizing surfaces (Steps (5), (6)), while Section 4 discusses the implementation of this construction using Fourier analysis, the construction of straight-field-line coordinates and the field-line Hamiltonian (Steps (7), (8)). Results using a typical magnetic field with islands from the PIES code [11] are presented in Section 5.

2. Quadratic flux minimizing coordinates

We begin with a representation of the magnetic field in toroidal coordinates and define the quadratic flux functional. Given field $\boldsymbol{B}$ in coordinates (ρ, θ, ϕ), with the contravariant components expressed as

$$B^\rho = \sum_{n,m} B^\rho_{n,m}(\rho) \sin(n\theta - m\phi) , \tag{1}$$

$$B^\theta = \sum_{n,m} B^\theta_{n,m}(\rho)\cos(n\theta - m\phi)\,, \tag{2}$$

$$B^\phi = \sum_{n,m} B^\phi_{n,m}(\rho)\cos(n\theta - m\phi)\,, \tag{3}$$

and the transformation to cylindrical coordinates (R,ϕ,z) as

$$R = R_{\rm maj} + \sum_{n,m} x_{n,m}(\rho)\cos(n\theta - m\phi)\,,$$

$$z = \sum_{n,m} y_{n,m}(\rho)\sin(n\theta - m\phi)\,. \tag{4}$$

This representation is consistent with the stellarator symmetry often used and discussed in detail in Ref. [12]. The Jacobian of the (ρ,θ,ϕ) coordinates, $\mathcal{J}_{\rho\theta\phi}$, is given by $\mathcal{J}_{\rho\theta\phi} = (\partial_\theta R\,\partial_\rho z - \partial_\rho R\,\partial_\theta z)R$.

Quadratic-flux-minimizing surfaces are defined [9,10] as the toroidal surfaces that minimize the quadratic flux φ_2,

$$\varphi_2 = \int_\Gamma \frac{B_n^2}{2C_n}\,{\rm d}\sigma\,, \tag{5}$$

where, for any vector field, the normal component $F_n \equiv \boldsymbol{F}\cdot\boldsymbol{n}$, with $\boldsymbol{n}$ being the unit normal to the trial surface Γ. In the above, $\boldsymbol{C}$ is an arbitrary divergence-free field, which we choose to be $\boldsymbol{C} = \nabla\theta\times\nabla\phi$ (a choice which is motivated by the action interpretation of the quadratic flux [10]). On allowing the surface to vary, we obtain the Euler–Lagrange equation for the extremal surface,

$$\boldsymbol{B}_\nu\cdot\nabla\nu = 0\,, \tag{6}$$

where $\nu \equiv B_n/C_n$ and $\boldsymbol{B}_\nu \equiv \boldsymbol{B} - \nu\boldsymbol{C}$ is called the *pseudo magnetic field*. In the action interpretation the parameter ν arises naturally as the functional derivative of the action, $\oint \boldsymbol{A}\cdot{\rm d}\boldsymbol{l}$, defined on a closed loop and thus we call it the *action gradient*.

Eq. (6) is a statement that ν is constant on a pseudo-field line. By focusing attention on *closed* pseudo-field lines, which are disjoint, we allow ν to have a different value on each line and thus can construct a toroidal quadratic flux minimizing surface by locating the periodic orbits of a continuous family of pseudo-field-line flows parameterized by ν (or equivalently, by the starting value of θ at $\phi = 0$). The additional field $\nu\boldsymbol{C}$ may be thought of as a correction to the real magnetic field chosen so as to cancel the radial component, thus producing periodic orbits at any given θ. Locating each pseudo orbit is a two-dimensional search – in ν and a suitable radial variable defining the initial position of the orbit (θ and $\phi = 0$ being held fixed).

Such periodic "pseudo" orbits are *true* closed magnetic field lines when the action gradient ν vanishes (a consequence of Hamilton's principle). In an integrable field, $\nu = 0$ for the entire family of periodic pseudo orbits making up the given rational rotational transform flux surface. In a generic, non-integrable system it is true only for the discrete set of action minimizing and minimax periodic orbits (X and O points in a Poincaré section) associated with the island chains.

As shown by Hudson and Dewar [5], ν provides an efficient measure of the size and phase of island chains. They introduced a method enabling the islands present in vacuum magnetic fields of stellarators to be manipulated. This technique allows vacuum configurations to be found with given islands set to arbitrary size and phase, and in particular allows islands to be removed without significantly altering the rotational transform profile.

Conversely, on a "good" magnetic surface, where the island width is already very small, $\nu \approx 0$ for all pseudo orbits and the ν search for a periodic pseudo orbit converges, to within any reasonable tolerance, in one iteration. Thus the search for each periodic orbit is effectively one-dimensional in regions where the surfaces are good. Hence the quadratic-flux-minimizing pseudo-orbit method does not in practice lead to a greater computational overhead than other field-line-tracing methods for constructing approximate magnetic surfaces in the good cases, while continuing to work efficiently and robustly when the surfaces are bad.

Having constructed a set of quadratic-flux-minimizing surfaces, $\{\rho = \rho_k(\theta,\phi)\}$, we label each surface by the value, s_k, of a coordinate s, which may be the toroidal flux, or any other good radial coordinate. Then we Fourier analyze in θ and ϕ, so that $\rho_k = \sum_{nm}\rho_{nm}(s_k)\cos(n\phi - m\theta)$. We define a coordinate transformation from ρ,θ,ϕ space to s,θ,ϕ space by interpolating the Fourier coefficients $\rho_{nm}(s_k)$ in the new variable s using cubic splines. With the transformation $\rho = \rho(s,\theta,\phi)$ thus determined, a standard vector transformation provides the new contravariant radial component $B^s \equiv \boldsymbol{B}\cdot\nabla s$, the toroidal and poloidal components $B^\phi \equiv \boldsymbol{B}\cdot\nabla\phi$ and $B^\theta \equiv \boldsymbol{B}\cdot\nabla\theta$ remaining invariant.

3. Construction of integrable magnetic field

Note that, by construction, $\nu = \mathcal{J}_{s\theta\phi}B^s$ on each quadratic-flux-minimizing surface $s = s_k$, where $\mathcal{J}_{s\theta\phi} \equiv (\partial_\theta R\,\partial_s z - \partial_s R\,\partial_\theta z)R \equiv 1/\boldsymbol{C}\cdot\nabla s$ is the Jacobian in the new coordinates. It is thus natural to extend the definition of ν to the whole domain by *defining* it by $\nu \equiv \mathcal{J}_{s\theta\phi}B^s$ everywhere. Then the divergence-free condition on $\boldsymbol{B}$ gives

$$\partial_s\nu + \partial_\theta(\mathcal{J}_{s\theta\phi}B^\theta) + \partial_\phi(\mathcal{J}_{s\theta\phi}B^\phi) = 0\,. \tag{7}$$

An integrable magnetic field, $\bar{\boldsymbol{B}}$, with invariant surfaces coinciding with the tori s = const, would have $\bar{B}^s \equiv 0$. We construct such a field from $\boldsymbol{B}$ by seeking a divergence-free correction field, $\delta\boldsymbol{B}$, such that $\mathcal{J}_{s\theta\phi}\delta B^s = -\nu$. If we can find such a field, then the total field $\bar{\boldsymbol{B}} \equiv \boldsymbol{B} + \delta\boldsymbol{B}$ will automatically satisfy the integrability condition $\mathcal{J}_{s\theta\phi}\bar{B}^s \equiv 0$.

An equation for $\delta\boldsymbol{B}$ is provided by the divergence-free condition

$$\partial_\theta(\mathcal{J}_{s\theta\phi}\delta B^\theta) + \partial_\phi(\mathcal{J}_{s\theta\phi}\delta B^\phi) = \partial_s\nu\,. \tag{8}$$

Since this is one equation for two unknowns, there is considerable arbitrariness in choosing δB^θ and δB^ϕ, but any convenient choice will be satisfactory for demonstrating the existence.

Unlike Eq. (7), where the two large terms in B^θ and B^ϕ must almost cancel to balance the small term in ν, we wish both δB^θ and δB^ϕ to be small, $\mathcal{O}(\nu)$. We proceed by choosing $\mathcal{J}_{s\theta\phi}\delta B^\phi$ to be constant with respect to θ and such as to balance the θ-average of $\partial_s\nu$,

$$\partial_\phi(\mathcal{J}_{s\theta\phi}\delta B^\phi) = \langle\partial_s\nu\rangle\,, \tag{9}$$

where $\langle\ \rangle$ denotes averaging with respect to θ over the interval 0 to 2π. Then Eq. (8) becomes

$$\partial_\theta(\mathcal{J}_{s\theta\phi}\delta B^\theta) = \partial_s\nu - \langle\partial_s\nu\rangle\,, \tag{10}$$

which may be integrated with respect to θ to produce a periodic solution because the choice equation (9) has been made so as to satisfy the solubility condition that the right-hand side of Eq. (10) has zero average with respect to θ. Of course one also needs to ask whether Eq. (9) can be integrated to give a periodic δB^ϕ, which requires the condition $\langle\langle\partial_s\nu\rangle\rangle = 0$ to be satisfied, where $\langle\langle\ \rangle\rangle$ denotes a double average with respect to both θ and ϕ. Inspection of Eq. (7) shows that $\partial_s\nu$ must satisfy this condition automatically.

4. Fourier implementation

To implement the above construction in practice, we first Fourier analyze

$$\nu = \sum_{n,m}\nu_{n,m}(s)\sin(n\phi - m\theta)\,, \tag{11}$$

$$\mathcal{J}_{s\theta\phi}B^\theta = \sum_{n,m}(\mathcal{J}_{s\theta\phi}B^\theta)_{n,m}(s)\cos(n\phi - m\theta)\,, \tag{12}$$

$$\mathcal{J}_{s\theta\phi}B^\phi = \sum_{n,m}(\mathcal{J}_{s\theta\phi}B^\phi)_{n,m}(s)\cos(n\phi - m\theta)\,. \tag{13}$$

Eqs. (9) and (10) then give

$$\left((\mathcal{J}_{s\theta\phi}\delta B^\theta)_{nm}, (\mathcal{J}_{s\theta\phi}\delta B^\phi)_{nm}\right) = \begin{cases} (0,0)\,, & n=0\,,\ m=0\,,\\ (\partial_s\nu_{n,m}/m, 0)\,, & n=0\,,\ m\neq 0\,,\\ (0, -\partial_s\nu_{n,m}/n)\,, & n\neq 0\,,\ m=0\,,\\ (\partial_s\nu_{n,m}/m, 0)\,, & n\neq 0\,,\ m\neq 0\,. \end{cases} \tag{14}$$

Then $\bar{\boldsymbol{B}} = (B^\theta + \delta B^\theta)\boldsymbol{e}_\theta + (B^\phi + \delta B^\phi)\boldsymbol{e}_\phi$ is a divergence-free field with flux surfaces s. As the surfaces s are those that minimize the quadratic flux functional at as many selected surfaces as desired, $\bar{\boldsymbol{B}}$ is an integrable field "as close as possible" to the given nearly integrable field. Note that additional freedom exists in the construction of the integrable field. Given that $\nabla\cdot\bar{\boldsymbol{B}} = 0$, and $\bar{\boldsymbol{B}}\cdot\nabla s = 0$, the poloidal and toroidal components may be deformed and a divergence free field maintained provided $m(\mathcal{J}_{s\theta\phi}\bar{B}^\theta)_{nm} = n(\mathcal{J}_{s\theta\phi}\bar{B}^\phi)_{nm}$. There is no freedom for the modes with either n or m being zero.

$\bar{\boldsymbol{B}}$ may be written in the Clebsch form [3, pp. 116–120] $\bar{\boldsymbol{B}} = \nabla s\times\nabla\lambda$, where $\lambda(s,\theta,\phi)$ may quite generally be written $\lambda = \dot\psi_{\rm t}(s)\theta - \dot\psi_{\rm p}(s)\phi + \tilde\lambda(s,\theta,\phi)$, where $\psi_{\rm t}, \psi_{\rm p}$ are the toroidal and poloidal flux functions and $\tilde\lambda$ is periodic in θ and ϕ, $\tilde\lambda = \sum\tilde\lambda_{nm}(s)\sin(n\phi - m\theta)$. The function $\tilde\lambda$ is determined

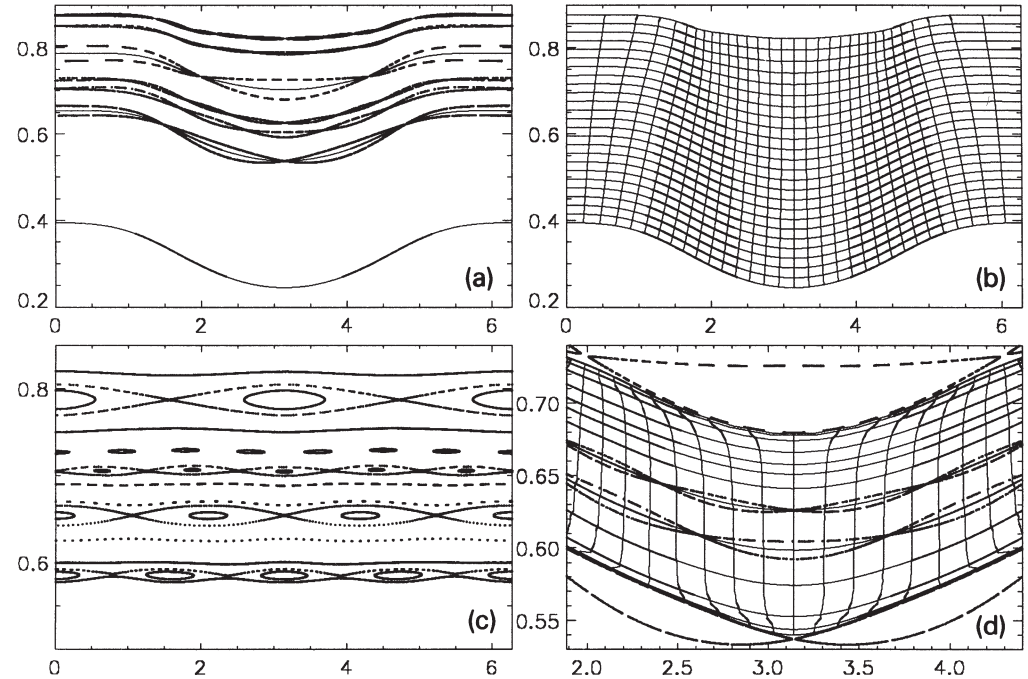

Fig. 1. (a, top left) Poincaré plot in arbitrary background coordinates and surfaces chosen for coordinate basis. (b, top right) Coordinate grid constructed from surfaces shown in 1a. (c, bottom left) The same Poincaré plot plotted in the new coordinates. (d, bottom right) Surfaces chosen for coordinate basis and new angle grid.

from $\partial_\theta \tilde{\lambda} = \mathcal{J}_{s\theta\phi}\bar{B}^\phi - \dot{\psi}_{\rm t}$ and $\partial_\phi \tilde{\lambda} = \dot{\psi}_{\rm p} - \mathcal{J}_{s\theta\phi}\bar{B}^\theta$. Averaging the first of these equations with respect to θ we see that $\dot{\psi}_{\rm t} \equiv \langle \mathcal{J}_{s\theta\phi} \rangle$, which implies that $\langle \mathcal{J}_{s\theta\phi}\bar{B}^\phi \rangle$ is independent of ϕ (i.e., $(\mathcal{J}_{s\theta\phi}\bar{B}^\phi)_{n0} = 0$ for $n \neq 0$). This is a consequence of conservation of flux through a narrow poloidal ribbon between surfaces s and $s+{\rm d}s$. Similarly, from the second equation, $(\mathcal{J}_{s\theta\phi}\bar{B}^\theta)_{0m} = 0$ for $m \neq 0$. The rotational transform obeys [3, p. 83, (4.8.4a)] $\dot{\psi}_{\rm p} = \dot{\psi}_{\rm t}\, \iota(s)$ where ι is the rotational transform. Thus $\iota = (\mathcal{J}_{s\theta\phi}\bar{B}^\theta)_{0,0}/(\mathcal{J}_{s\theta\phi}\bar{B}^\phi)_{0,0}$.

Rather than integrate the o.d.e.'s for the solution of $\tilde{\lambda}$, we may utilize the Fourier representation to write

$$\tilde{\lambda}_{nm} = \begin{cases} -(\mathcal{J}_{s\theta\phi}\bar{B}^\phi)_{nm}/m\,, & m \neq 0\,, \\ -(\mathcal{J}_{s\theta\phi}\bar{B}^\theta)_{nm}/n\,, & n \neq 0\,, \end{cases} \tag{15}$$

where the $(0,0)$ mode can be ignored. The straight-field-line angle for the integrable field is then obtained by absorbing the periodic function $\tilde{\lambda}$ into the poloidal angle $\theta_0 = \theta + \tilde{\lambda}/\dot{\psi}_{\rm t}$. The original magnetic field may be written in the straight-field-line coordinates for the constructed integrable magnetic field by a standard vector transformation and the new Jacobian $\mathcal{J}_{s\theta_0\phi} = \mathcal{J}_{s\theta\phi}\partial_{\theta_0}\theta$ is obtained.

The coordinates thus constructed enable the original magnetic field to be expressed in terms of a canonical, nearly integrable, field line Hamiltonian, χ, such that

$$\boldsymbol{B} = \nabla s \times \nabla\theta_0 + \nabla\phi \times \nabla\chi(s, \theta_0, \phi)\,, \tag{16}$$

where $\chi = \psi_{\rm p}(s) + \sum_{nm} \chi_{nm}(s) \cos(n\phi - m\theta_0)$. The Fourier components of the magnetic field in (s, θ_0, ϕ) coordinates are simply related to the Fourier components of the field line Hamiltonian through

$$\begin{aligned} (\mathcal{J}_{s\theta_0\phi}B^s)_{nm} &= -m\chi_{nm}\,, \\ (\mathcal{J}_{s\theta_0\phi}B^{\theta_0})_{nm} &= \dot{\chi}_{nm}\,. \end{aligned} \tag{17}$$

It is convenient to use the field line Hamiltonian representation, as this form guarantees the divergence free property of the magnetic field. The field line Hamiltonian provides all information regarding the magnetic field. The (n,m) island width Δ_{nm} may be estimated in the thin island approximation using $\Delta_{nm} = (\chi_{nm}/\iota'_{nm})^{1/2}$ where χ_{nm} and ι'_{nm} are the (n,m) mode amplitude and the shear at the resonant surface, respectively.

5. Application and discussion

A trial magnetic field is obtained from PIES [11]. To construct straight-field-line coordinates for the assumed underlying integrable magnetic field, a small set of low order rational rotational transform surfaces is selected. These pass directly through the major resonances and are likely to be least deformed by the assumed small perturbation. A Poincaré plot of the surfaces chosen and the major island chains is shown in Fig. 1a. The constructed coordinate grid is shown in Fig. 1b. The Poincaré section in the new coordinates is shown in Fig. 1c.

To construct straight-field-line coordinates for the given magnetic field, as much as that is possible given the islands, we may choose a set of high order rational rotational transform surfaces that lie just outside the largest separatrices. A Poincaré plot showing the island chains and the surfaces chosen as the coordinate foundation and the constructed angle grid is given in Fig. 1d. Note that the closer the quadratic flux surfaces trace out the separatrices, the more singular the coordinates constructed become. This is due to the fact that straight-field-line coordinates are not consistent with the existence of separatrices.

The surfaces may be chosen such that the periodicities approximate noble irrationals using truncations of the continued fraction representations of these numbers. This being the case, the quadratic flux minimizing surfaces will approximate KAM surfaces and enable a partition of phase space into chaotic and regular regions. Chaos will help to lessen the singularity of the straight-field-line coordinates near separatrices by providing a limit on how close the separatrix can be approached.

The coordinate construction is flexible and robust. The type of coordinates constructed depend solely on the choice of the rationals that identify the quadratic flux minimizing surfaces to be used as the coordinate framework. Various types of coordinates may be constructed. For example, straight-field-line coordinates for a nearby integrable magnetic field or straight-field-line coordinates for the given magnetic field in regions of good surfaces. With a careful choice of surfaces that approximate KAM surfaces, coordinates that partition regions of chaos also may be constructed. Also provided by the method is the location of X and O points of the island chains, and the field line Hamiltonian in a convenient nearly integrable form. No assumptions need be made regarding the structure of the magnetic field. Finally, the procedure is applicable to nearly integrable Hamiltonian systems, such as in accelerator theory and celestial dynamics [13].

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